DEGREE & DIPLOMA ENGINEERING

MATHEMATICS SOLUTION
(CBCGS SEM – 4 DEC 2018)
BRANCH – IT ENGINEERING

1a) The equations of lines of regression are x +2y = 5 and 2x+3y=-8. (5)
Find (i)means of x and y, (ii)coefficient of correlation between x and y.

Ans. lines of regression are x + 2y = 5 and 2x+3y= -8.

$\therefore y = -\frac{1}{2}x + \frac{5}{2} \longrightarrow (1)\ and \quad y = -\frac{2}{3}x - \frac{8}{3} \longrightarrow (2)$

Let $b_1 = -\frac{1}{2}$ and $b_2 = -\frac{2}{3}$

Since $|b_1| < |b_2|$, $b_{xy} = b_1 = -\frac{1}{2} = -\frac{3}{2} \longrightarrow (3)$

Hence, equation (1) is regression equation of y and x type and equation (2) is regression equation of X and Y type.

From (1) and (2), $-\frac{1}{2}x + \frac{5}{2} = -\frac{2}{3}x - \frac{8}{3}$

$\therefore \frac{2}{3}x - \frac{1}{2}x = -\frac{8}{3} - \frac{5}{2}$

$\therefore \frac{1}{6}x = -\frac{31}{6}$

$\therefore x = -31 \quad \therefore \bar{x} = -31$

Substituting x = -31 in (2), $y = -\frac{2}{3}(-31) - \frac{8}{3}$

$\therefore y = 18 \quad \therefore \bar{y} = 7$

Now, $r = \pm\sqrt{b_{yx} . b_{xy}} \times$

$= \pm\sqrt{\frac{-1}{2} \times \frac{-3}{2}}$ (from3)

$= \pm\ 0.8660$

Since, b_{yx} and b_{xy} are both negative, 'r' is negative. $\therefore$ r = -0.8660

1b) Show that $97|(2^{48} - 1)$. **(5)**

Ans. We know, $2^{11} = 2048 = 97 \times 21 + 11$

$\therefore 2^{11} \equiv 11 \pmod{97}$

$\therefore (2^{11})^4 \equiv 11^4 \pmod{97} \longrightarrow (1)$

But, $11^4 \equiv 14641 = 97 \times 151 - 6$

$\because 11^4 \equiv (-6) \pmod{97} \longrightarrow (2)$

From (1) and (2), $(2^{11})^4 \equiv (-6) \pmod{97}$

$\therefore 2^{44} \equiv (-6) \pmod{97}$

Multiply both sides by 2^4 ,

$\therefore 2^{44} \times 2^4 \equiv (-6 \times 2^4) \pmod{97}$

$\therefore 2^{48} \equiv -96 \pmod{97}$

$\therefore 2^{48} \equiv (-96 + 97) \pmod{97}$

$\therefore 2^{48} \equiv 1 \pmod{97}$

$\therefore 97 | (2^{48} - 1)$

1c) The probability density function of a random variable X is zero except at x =0,1,2 and p(0) = $3k^3$, p(1) = $4k - 10k^2$, p(2) = $5k - 1$. Find (i) k and (ii) $p(0 < X \leq 2)$. **(5)**

Ans. For any probability mass function, $\sum_{i=-\infty}^{\infty} p_1 = 1$

$\therefore p(0) + p(1) + p(2) = 1$

$\therefore 3k^3 + (4k - 10k^2) + (5k - 1) = 1$

$\therefore 3k^3 + 4k - 10k^2 + 5k - 2 = 0$

$\therefore 3k^3 - 10k^2 + 9k - 2 = 0$

On solving, we get, $k = 1, 2, \frac{1}{3}$

But, $0 \leq P_1 \leq 1$

When k = 1, $p(0) = 3(1)^3 = 3$, which is not possible.

When k = 2, $p(0) = 3(2)^3 = 24$, which is not possible.

$\therefore k = \frac{1}{3}$

$\therefore P(0) = 3 \times \frac{1}{3^3} = \frac{1}{9}$

$\therefore p(1) = 4k - 10k^2 = 4\left(\frac{1}{3}\right) - 10\left(\frac{1}{3}\right)^2 = \frac{2}{9}$

$\therefore p(2) = 5k - 1 = 5\left(\frac{1}{3}\right) - 1 = \frac{2}{3}$

The p.m.f is

X	0	1	2
P(x – x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{3}$

$\therefore p(x<1) = p(0) = \frac{1}{9}$

$\therefore p(0< X \le 2) = p(1) + p(2) = \frac{1}{9} + \frac{2}{9} = \frac{1}{3}$

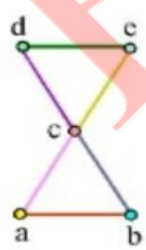
Hence,

$K = \frac{1}{3}$; $p(x < 1) = \frac{1}{9}$ and $p(0< X \le 2) = \frac{1}{3}$.

1d) Give an example of a graph which has **(05)**

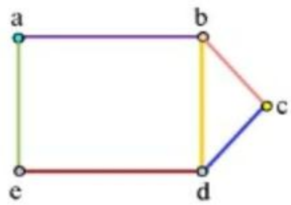
Ans:

(i) Eulerian circuit but not a Hamiltonian circuit



All the vertices are of even degree. Hence by theorem there is Eulerian circuit.
Eulerian circuit : abcdeca
The circuit is not Hamiltonian because there is no circuit which contains all the vertices only once.

(ii) Hamiltonian circuit but not an Eulerian circuit



All the vertices can be traversed only once. Hence there is Hamiltonian circuit.
Hamiltonian circuit : abcdea
The degree of vertices b & d are odd. Hence there is no Eulerian circuit.

2a) Find gcd (2947, 3997) using Euclidean Algorithm. Also find x and y such that 2947x + 3997y = gcd (2947,3997). **(06)**

Ans. Part I : Let a = 2947 and b = 3997

Using Euclid Algorithm.

1)	$3997 = 1 \times 2947 + 1050$	$\therefore b = 1 \times a + 1050$ $\therefore b - a = 1050$
2)	$2947 = 2 \times 1050 + 847$	$\therefore a = 2 \times (b - a) + 847$ $\therefore a = 2b - 2a + 847$ $\therefore 3a - 2b = 847$
3)	$1050 = 1 \times 841 + 203$	$\therefore b - a = 1 \times (3a - 2b) + 203$ $\therefore b - a = 3a - 2b + 203$ $\therefore 3b - 4a = 203$
4)	$847 = 4 \times 203 + 35$	$\therefore 3a - 2b = 4 \times (3b - 4a) + 35$ $\therefore 3a - 2b = 12b - 16a + 35$ $\therefore 19a - 14b = 35$
5)	$203 = 5 \times 35 + 28$	$\therefore 3b - 4a = 5 \times (19a - 14b) + 28$ $\therefore 3b - 4a = 95a - 70b + 28$ $\therefore 73b - 99a = 28$
6)	$35 = 1 \times 28 + 7$	$\therefore 19a - 14b = 1 \times (73b - 99a) + 7$ $\therefore 19a - 14b = 73b - 99a + 7$ $\therefore 118a - 87b = 7 \longrightarrow (8)$
7)	$8 = 4 \times 7 + 0$	

$\therefore$ gcd (2947, 3997) = 7

Part II:

Given, 2947x + 3997y = ged (2947, 3997)

∴ ax + by = 7 ⟶ (9)

Comparing (8) & (9), x = 118 and y = -87

i.e, x_0 = 118 and y_0= -87 is the solution of 2947x + 3997y = ged(2947, 3997)

other solution are $x = x_0 + \left(\frac{b}{d}\right) t \; and \; y = y_{0-} \left(\frac{a}{d}\right) t$ where 't' is arbitrary & d = ged of a & b i.e, d = (a,b)

∴ x = 118 + $\left(\frac{3997}{7}\right) t \; and \; y = -87 - \left(\frac{2947}{7}\right) t$

∴ Other solutions are $x = 118 + 571t \quad and \quad y = -87 - 421t$

2b) The four roots of unity G = (1 , -1 , i , -i) forms a group under multiplication. (06)

Ans. Let a, b ∈ G

The composition table is

*	1	−1	i	−i
1	1	−1	i	−i
−1	−1	1	−i	i
i	i	−i	−1	1
−i	−i	i	1	−1

From above table, we observe,

a * b exists and a * b∈ G.

∴ *is binary operator in G.

G1:

Multiplication of complex number is associative.

∴ * is associative.

G2:

From table, we observe, first row is same as the header.

∴ 1 ∈ G is the identity.

∴ identify exists.

G3:

From table, we observe, identify elements (i.e,1) is present in each row.

∴ $1^{-1} = 1$; $(-1)^{-1} = -1$; $i^{-1} = -i$; $(-i)^{-1} = i$

∴ Inverse of each element exist and each inverse ∈ G.

∴ Inverse exists.

Hence, G is group usual multiplication of complex number.

2c) Find whether the following graphs G = (v, E) and G' = (v',E') are isomorphic? Justify. (8)

(1)v = {a, b, c, d} E = { (a, b), (a,d), (b,d), (c,a), (c,b), (d,c)}

(2)v' = { 1, 2, 3, 4}, E' = { (1,2), (2,3), (3,1), (3,4), (4,1), (4,2)}.

Ans. Definition :

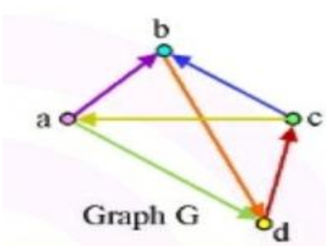
Two graphs G (v, E) & G' (v' , E') are isomorphic if

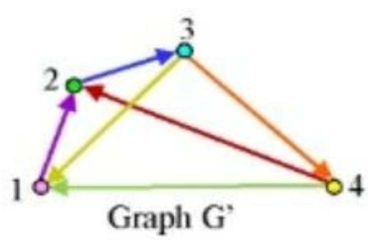
1) Number of edge between v_1 &v_2 is same as the number of edges between v_1'&v_2'.

2)G and G' have equal number of vertices.

3) G and G' have equal number of edges.

4) Adjacency property is observed





From the above two graphs

Graph G		
Number of vertices	4	
Number of Edges	6	
Vertex	Degree*of Vertex	Adjacent Vertices (Degree* in bracket)
a	2	b (1), d (1)
b	1	d (1)
C	2	a (2), b (1)
d	1	C (2)

GRAPH ‘G’		
Number of vertices	4	
Number of Edges	6	
Vertex	Degree* of Vertex	Adjacent Vertices (Degree* in bracket)
1	1	2(1)
2	1	3(2)
3	2	1(1),4(2)
4	2	1(1),2(1)

We observe in both the graphs, there are

1) Equal number of vertices.
2) Equal number of edges.
3) Four vertices with degree 3.
4) Adjacency property is observed.

Hence, the given two graphs are isomorphic.

3a) Show that (D_8, $\leq$) is a lattice. Draw its Hasse diagram. (6)

Ans. D_8 means divisors of 8.

D_8 = { 1,2,4,8}

The Hasse diagram of R is as shown

We know the relation of divisibility is a partial order relation.

∴ Set (D_8, $\leq$) is a poset.

The composition table of LUB (least upper bound) and GLB (greater lower bound) are,

∨	1	2	4	8
1	1	2	4	8
2	2	2	4	8
4	4	4	4	8
8	8	8	8	8

∧	1	2	4	8
1	1	1	1	1
2	1	2	2	2
4	1	2	4	4
8	1	2	4	8

From the above two tables, we observe that every pair of elements of D_8 has LUB (latest upper bound) and GLB (greatest lower bound)

Also, each LUB and GLB ∈ D_8

Hence, (D_8,$\leq$) is a lattice.

3b) The local authorities in a certain city install 10,000 electric lamps in the streets of the city. If these lamps have an average life of 1000 burning hours with a standard deviation of 200 hours, how many lamps might be expected to fail (i)in the first 8000 burning hours? (ii) Between 800 and 1200 burning hours? (6)

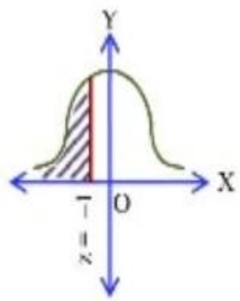
Ans. Mean (m) = 10000

Standard deviation (σ) = 200

N = 10000

Let X denote the burning life of the electric lamp.

(i)P (lamps fail in the first 800 burning hours) = P (X < 800)



= P $\left(\frac{x-m}{\sigma} < \frac{800-1000}{200}\right)$

= p (z < -1)

= 0.5 – Area between 'Z = 0' to 'Z = -1'

= 0.5 – 0.3413

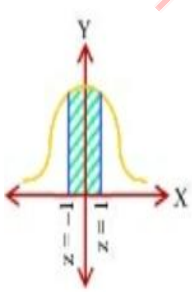
= 0.1587

∴ Number of lamps failing in the first 800 burning hours = N x P(x < 800)

= 10000 x 0.1587

= 1587 lamps

(iii) P (lamps fail between 800 and 1200 burning hours) = P (800<X<1200)



$= P\left(\frac{800-1000}{200} < \frac{X-m}{\sigma} < \frac{1200-1000}{200}\right)$

= P (-1< z <1)

= Area between 'z = 0' to 'z = -1' + Area between 'z = 0' to 'z = 1'

= 0.3413 + 0.3413

= 0.6826

∴ Number of lamps failing between 800 and 1200 burning hours

= N x P(800<X<1200)

= 10000 x 0.6826

= 6826 lamps

3c) Find inverse of 2^{-1}(mod31) using Fermat's theorem. (4)

Ans. 31 is a prime number

Let a = 2 and p = 31 , We observe $p \nmid a$

By Fermat's little theorem, a^{-1} (mod p) ≡ a^{P-2} (mod p)

∴ 2^{-1}(mod31) ≡ 2^{29}(mod31) -------(1)

We know, 2^5 = 32 = 1 x 31 +1

∴ 2^5 ≡ 11(mod31)

∴ $(2^5)^5$ ≡1^5 (mod31)

∴ 2^{25}≡ 1(mod31)

Multiply both sides by 2^4 .

∴ 2^{25} x 2^4≡ (1x2^4) (mod31)

∴ 2^{29}≡ 16(mod31) ------(2)

∴ 2^{-1}(mod31) = 16. (from 1 & 2)

3d) Find the legendre's symbol of $\left(\frac{19}{23}\right)$. **(4)**

Ans. $\left(\frac{19}{23}\right)$

Let a = 19 and p = 23, which is an odd prime.

We observe $p \nmid a$.

By Euler's critertion, $\left(\frac{a}{p}\right) \equiv a^{(p-1)/2} \pmod p$

$\therefore \left(\frac{19}{23}\right) \equiv 19^{(23-1)/2} (\text{mod}23)$

$\therefore \left(\frac{19}{23}\right) \equiv 19^{11} (\text{mod}23) \longrightarrow (1)$

Now, $19 \equiv -4(\text{mod}23)$

$\therefore 19^2 \equiv (-4)^2(\text{mod}23)$

$\therefore 19^2 \equiv 16(\text{mod}23)$

$\therefore 19^2 \equiv (-7)(\text{mod}23)$

$\therefore (19^2)^5 \equiv (-7)^5$ (mod23

But, $(-7)^5 = -16807 = -730 \times 23 - 17$

$\therefore 19^{10} \equiv -17(\text{mod}23)$

$\therefore 19^{10} \equiv 6\ (\text{mod}23)$

Multiply both side by 19,

$\therefore 19^{10} \times 19 \equiv (6 \times 19)(\text{mod}23)$

But, $6 \times 19 = 114 = 5 \times 23 - 1$

$\because 19^{10} \equiv (-1)(\text{mod}23) \longrightarrow (2)$

From (1)&(2), $\left(\frac{19}{23}\right) \equiv -1$, i.e, 19 is not a quadratic residue modulo 23.

4a) Calculate the coefficient of correlation between x and y from the following data . (6)

x	**12**	**9**	**8**	**10**	**11**	**13**	**7**
y	**14**	**8**	**6**	**9**	**11**	**12**	**3**

Ans. let X and Y denote height of father and height of son respectively.

X	Y	$u_{i=} x_i - 10$	$v_i = y_i - 9$	${u_i}^2$	${v_i}^2$	$u_i v_i$
12	14	2	5	4	25	10
9	8	-1	-1	1	1	1
8	6	-2	-3	4	9	6
10	9	0	0	0	0	0
11	11	1	2	1	4	2
13	12	3	3	9	9	9
7	3	-3	-6	9	36	18
	Total	0	0	28	84	46

Here, n= 7

Karl pearson's coefficient of correlation $r_{x,y} = r_{u,v} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$

$$= \frac{7(46) - (0)(0)}{\sqrt{7(28) - (0)^2}\sqrt{7(84) - (0)^2}}$$

$$= \frac{322}{\sqrt{196}\sqrt{588}}$$

$$= 0.9485$$

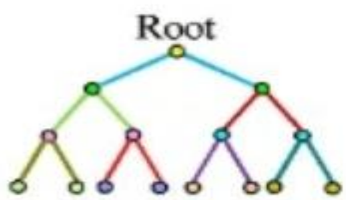
Hence, coefficient of correlation between height of father and height of son(r)=0.9485.

4b) Draw a connected graph for which every edge is a cut edge. (3)

Ans. A cut edge set is a set of edges of a graph which, if removed(or ''cut''), disconnects the graph(i.e, forms a disconnected graph). OR A cut – edge is an edge, which when removed increases the number of components.

A connected graph for which every edge is a cut edge is a TREE.

Graph (or Tree) in which every edge is a cut edge is as shown



Full Binary Tree

4c) Show that any connected graph with 'n-1' edges is a tree. (3)

Ans. A tree is a connected graph without any cycles, or a tree is a connected non-cyclic graph.

Let T be a connected graph without any cycles.

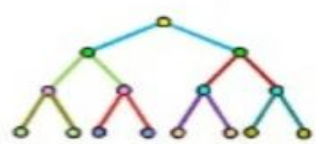
Let n and 'e' be the number of vertices and edges in T.

Let ''e = n-1''.

For n = 1, e = 1-1 = 0 edge. i,e There is no edge which connects a vertex.

For n = 2, e =2 -1 = 1 edge. i.e, One edge connects two vertices.

For n = 3, e = 3-1 = 2 edges. i.e, Two edges connecting three vertices will not be cycle so it is a tree.



Tree T

4d) Can it be concluded that the average life span of an Indian is more than 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with S.D of 8.9 years. **(4)**

Ans. n = 100(> 30, so it is large sample)

$\bar{x} = 71.8$; $s = 8.9$

Step 1:

Null Hypothesis (Ho) : $\mu = 70$ (i.e, the average life span of an Indian is 70 years)

Alternative Hypothesis (H_a) : $\mu > 70$ (i.e, the average life span of an Indian is more than 70 year) (one tailed test)

Step 2:

Level of significance (LOS) = 5% (Two tailed test)

LOS = 10% (one tailed test)

∴ Critical value (Z_a) = 1.64

Step 3:

Since sample is large,

$$S\,E. = \frac{s}{\sqrt{n}} = \frac{8.9}{\sqrt{100}} = 0.89$$

Step 4:

Test statistic

$$z_{cal} = \frac{\bar{x} - \mu}{S.E} = \frac{71.8 - 70}{0.89} = 2.0225$$

Step 5:

Since $z_{cal} > z_a$ H_o is rejected.

∴ The average life span of an Indian is more than 70 years.

4e) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 70, 71, 70 inches.
Discuss the suggestion that the mean height of population in 65 inches. (4)

Ans. n = 10 (< 30, so it is small sample)

Step 1:

Null Hypothesis (H_0) : $\mu = 65$ (i.e, the mean height of the population is 65 inches)

Alternative Hypothesis (H_0) : $\mu \neq 65$ (i.e, the mean height of the population is not 65 inches) (Two tailed test)

Step 2:

LOS = 5%(Two tailed test)

Degree of Freedom = n – 10 = 10 – 1 = 9

∴ Critical value (t_a) = 2.2622

Step 3:

Values (x_i)	$d_i = x_i - 67$	${d_i}^2$
63	-4	16
63	-4	16
64	-3	9
65	-2	4
66	-1	1
69	2	4
69	2	4
70	3	9
70	3	9
71	4	16
Total	0	88

$$\bar{d} = \frac{\Sigma d_i}{n} = \frac{0}{10} = 0$$

$\therefore \bar{x} = a + \bar{d} = 67 + 0 = 67$

Since sample is small, $s = \sqrt{\frac{\Sigma {d_i}^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$

$$= \sqrt{\frac{88}{10} - \left(\frac{0}{10}\right)^2}$$

$= 2.9965$

$$\text{S.E.} = \frac{s}{\sqrt{n} - 1}$$

$$= \frac{2.9965}{\sqrt{9}}$$

$= 0.98888$

Step 4: Test Statistic

$$t_{cal} = \frac{\bar{x} - \mu}{S.E}$$

$$= \frac{67 - 65}{0.98888}$$

$= 2.0227$

Step 5: Decision

Since $|t_{cal}| < t_a$, H_0 is accepted.

∴ The mean height of the population is 65 inches.

5a) Solve $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$. **(6)**

Ans. $x \equiv 5 \pmod 6$ let $a_1 = 5$ and $m_1 = 6$

$x \equiv 4 \pmod{11}$ let $a_2 = 4$ and $m_2 = 11$

$x \equiv 3 \pmod{17}$ let $a_3 = 3$ and $m_3 = 17$

Let $M_1 = m_2 \times m_3 = 11 \times 17 = 187$

$M_2 = m_1 \times m_3 = 6 \times 17 = 102$

$M_3 = m_1 \times m_2 \times m_3 = 6\text{x}11\text{x}17 = 1122$

Let $M_1x \equiv 1(\text{mod } m_1)$

$\therefore\ 187x \equiv 1(\text{mod}3) \longrightarrow (1)$

$\therefore\ 1 \equiv 187x\ (\text{mod}3)$

$\therefore 1 \equiv (62x \times 3 + x)(\text{mod}3)$

$\therefore 1 \equiv x(\text{mod}3)$

$\therefore x \equiv 1(mod3)$

$\therefore x_1 = 1$

Similarly,

Let $M_2x \equiv 1(\text{mod } m_2)$

$\therefore 102x \equiv 1(\text{mod}11) \longrightarrow (2)$

$\therefore\ 1 \equiv 102x\ (\text{mod}11)$

$\therefore\ 1 \equiv (9x \text{ x } 11 + 3x)(\text{mod}11)$

$\therefore\ 1 \equiv 3x\ (\text{mod}11)$

$\therefore\ 4 \equiv 12x(\text{mod}11)$

$\therefore 4 \equiv (1x \text{ x } 11 + x)(\text{mod}11)$

$\therefore 4 \equiv x(\text{mod}11)$

$\therefore\ x \equiv 4(mod11)$

$\therefore x_2 = 4$

Similarly,

Let $M_3x \equiv 1\ (\text{mod} m_3)$

$\therefore 66x \equiv 1(\text{mod}17) \longrightarrow (3)$

$\therefore 1 \equiv 66x\ (\text{mod}17)$

$\therefore\ 1 \equiv (4x \times 17 - 2x)(\text{mod}17)$

$\therefore 1 \equiv -2x\ (mod17)$

$\therefore\ -8 \equiv\ 16x(mod17)$ (multiply both side by – 8)

$\therefore -8 \equiv\ (1x\ \times 17 - x)(\text{mod}17)$

$\therefore -8 \equiv\ -x(\text{mod}17)$

$\therefore x\ \equiv\ 8(\text{mod}11)$

$\therefore\ x_3 = 8$

By Chinese Remainder Theorem, the solution of the given problem is

$x\ \equiv\ (a_1 M_1 x_1 + a_2 M_2 x_2 + a_3 M_3 x_3)(\text{mod M})$

$\therefore\ x \equiv\ (5 \times 187 \times\ 1 + 4 \times 102 \times 4 + 3 \times 66 \times 8)(\text{mod}1122)$

$\therefore\ x \equiv\ 4151(\text{mod}1122)$

$\therefore x \equiv\ (3 \times 1122 + 785)(\text{mod}1122)$

$\therefore x \equiv\ 785(\text{mod}1122)$

x = 785 is one solution . General solution is given by, x = 785+1122 k where k is any integer

5b) Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9: 3: 3: 1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Using chi – square verify does the experimental results support the theory? (6)

Ans. N = 1600;

Step 1;

Null Hypothesis (H_0): Distribution of beans is as per theory.

Alternative Hypothesis (H_a) : Distribution of beans is not as per theory.

Step 2:

Level of significance (LOS) = 5%

Degree of Freedom = n- 1 = 4-1 = 3

$\therefore$ Critical value ($x_a{}^2$) = 7.815

Step 3: Test Statistic.

Observed Frequency (0)	Expected Frequency(E)	$x^2 = \frac{(0-E)^2}{E}$
882	$\frac{9}{16}$ x $1600 = 900$	0.36
313	$\frac{3}{16}$x $1600 = 300$	0.56
287	$\frac{3}{16}$ x $1600 = 300$	0.56
118	$\frac{1}{16}$ x $1600 = 100$	3.24
	Total	4.72

$x_{cal}{}^2 = \Sigma\frac{(0-E)^2}{E} = 4.72$

Step 4: Decision

Since $x_{cal}{}^2 < xa^2, H_0$ is accepted.

∴ Distribution of beans is as per theory i.e, in the ratio 9:3:3:1.

5c) Let G be a group of all permutations of degree 3 on 3 symbols 1, 2 & 3. Let H={I (1, 2)} be a subgroup of G. Find all the distinct left cosets of H in G and hence index of H. (8)

Ans. G be a group of all permutations of degree 3 or 3 symbols 1, 2 & 3

∴Order of G = $|G| = 3! = 6$

Given, H = {I, (1, 2)} is a subgroup of G.

∴ Order of H = $|H| = 2! = 2$

By Lagrange's Theorem, Index = Number of different left cosets of subgroup H= $\frac{|G|}{|H|} = \frac{6}{2} = 3$

Consider, Left coset of (13)H = (13) { I,(1,2) } = { (13)I,(13)(12)} Now,

(13) I = (13)(since, I is the identity)

(13) (12) = (123)

∴ (13)H = { (13),(123)}

Similarly,

(23) H = (23) { I,(1 2)} = { (23)I,(23)(12)}

Now,

(23)I = (23) (since, I is the identity)

(23)(12) = (132)

∴(23)H = { (23),(132)}

Hence, the three distinct cosets of H are H, (13)H, (23)H

Index = 3

6a) Show that $53^{103} + 103^{53}$ is divisible by 39. **(6)**

Ans. We know, $53^2 = 2809 = 72 \times 39 + 1$

$\therefore 53^2 \equiv 1 \pmod{39}$

$\therefore (53^2)^{51} \equiv 1^{51} \pmod{39}$

$\therefore 53^{102} \equiv 1 \pmod{39}$

Multiply both side by 53,

$\therefore 53^{102} \times 53 \equiv 1 \times 53 \pmod{39}$

$\therefore 53^{103} \equiv 53 \pmod{39}$

$\therefore 53^{103} \equiv (1 \times 39 + 14) \pmod{39}$

$\therefore 53^{103} \equiv 14 \pmod{39} \longrightarrow (1)$

Now, $103^3 = 10609 = 272 \times 39 + 1$

$\therefore\ 103^2 \equiv 1 (mod 39)$

$\therefore\ (103^2)^{26} \equiv 1^{26} (mod 39)$

$\therefore\ 103^{52} \equiv 1 (mod 39)$

Multiply both sides by 103,

$\therefore\ 103^{52} \times 103 \equiv 1 \times 103 (mod 39)$

$\therefore\ 103^{53} \equiv 103 (mod 39)$

$\therefore\ 103^{53} \equiv (2 \times 39 + 25)(mod 39)$

$\therefore\ 103^{53} \equiv 25 (mod 39) \longrightarrow (2)$

Adding (1) & (2)

$53^{103} + 103^{53} \equiv (14 + 25)(mod 39)$

$\therefore\ 53^{103} + 103^{53} \equiv 39 (mod 39)$

$\therefore\ 53^{103} + 103^{53} \equiv (1 \times 39 + 0)(mod 39)$

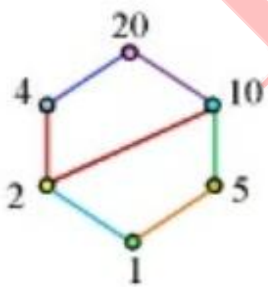
$\therefore\ 53^{103} + 103^{53} \equiv 0 (mod 39)$

$\therefore\ 53^{103} + 103^{53}$ is divisible by 39.

6b) Given L = { 1, 2, 4, 5, 10, 20} with divisibility relation. (6)

Verify that (L, ≤) is a distributive but not complimented Lattice.

Ans. The Hasse Diagram for L is



Consider $4 \vee 5 = 20$

$\therefore\ 2 \wedge (4 \vee 5) = 2 \wedge 20 = 2$

Also,

$2 \wedge 5 = 1$ & $2 \wedge 4 = 2$,

$\therefore\ (2 \wedge 4) \vee (2 \wedge 5) = 2 \vee 1 = 2$

$\therefore\ 2 \wedge (4 \vee 5) = (2 \wedge 4) \vee (2 \wedge 5)$

$\therefore$ Distributive property is satisfied.

Consider $5 \wedge 10 = 5$

$\therefore\ 4 \vee (5 \wedge 10) = 4 \vee 5 = 20$

Also,

$4 \vee 5 = 20$ & $4 \vee 10 = 20$,

$\therefore\ (4 \vee 5) \wedge (4 \vee 10) = 20 \vee 20 = 20$

$\therefore\ 4 \vee (5 \wedge 10) = (4 \vee 5) \wedge (4 \vee 10)$

$\therefore$ Distributive property is satisfied

$\therefore$ $(L, \leq)$ is a distributive Lattice.

(ii) By definition of a complement, $a \vee \bar{a} = 1$ and $a \wedge \bar{a} = 0$ i.e, $a \vee \bar{a} = 20$ & $a \wedge \bar{a} = 1$

The complements of elements of set L, are

Element	1	2	4	5	10	20
complement	20	__	5	4	_	1

$\therefore$ Complement of each element do not exists.

$\therefore$ $(L, \leq)$ is not a complemented Lattice.

6c) Write the following permutation as the product of disjoint cycles. F=(1 2) (1 2 3) (1 2).

(4)

Ans. Given, $f = (1\ 2)\ (1\ 2\ 3)(1\ 2)$

$f(1) = (1\ 2)\ (1\ 2\ 3)\ (1\ 2)\ (1)$

$= (1\ 2)\ (1\ 2\ 3)\ (2)$

$= (1\ 2)\ (3)$

$\therefore\ f(1) = (3)$

$f(2) = (1\ 2)\ (1\ 2\ 3)(\ 1\ 2)(2)$

$= (1\ 2)\ (1\ 2\ 3)\ (1)$

$= (1\ 2)(2)$

$\therefore\ f(2) = 1$

$f(3) = (\ 1\ 2\ )\ (1\ 2\ 3\ )(1\ 2)\ (3)$

$= (1\ 2\ )\ (1\ 2\ 3\ )\ (3)$

$= (1\ 2)\ (1)$

$\therefore\ f\ (3) = 2$

$$\therefore f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Hence, expressing permutation f as the product of disjoint cycles we have

$$f = (1\ 3\ 2)$$

6d) Express the expression (x+y) (x+z)(x'y') in the sum of product form. (4)

Ans. Consider,

$(x + y)(x + z)(x'y)'$

$\equiv [(x + y)(x + z)][(x')' + y']$ (De Morgan Law)

$\equiv [(x + y)(x + z)][x + y']$ (Involution Law)

$\equiv [(x + z)(x + y)](x + y')$ (commutative Law)

$\equiv (x + z)[(x + y)(x + y')]$ (Associative Law)

$\equiv (x + z) [x + (y y')]$ (Distributive Law)

$\equiv (x + z) [x + 0]$ (complement Law)

$\equiv (x + z) [x]$ (Identity Law)

$\equiv (xx + zx)$ (Distributive Law)

$\equiv (x + zx)$ (Idempotent Law)

Hence, $(x + y) (x + z) (x'y)' \equiv (x + zx)$, which is the sum- of – product form.